

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY GURAJADA VIZIANAGARAM

I B. Tech II Semester Supplementary Examinations Dec – 25/ Jan - 26

Differential Equations and Vector Calculus

(Common to all branches)

Time: 3 hours

Max. Marks: 70

*Question paper consists of Part A & Part B.
Part A is compulsory, Answer all questions.
In Part B, Answer any one question from each unit.

PART-A

(20 Marks)

- 1 a) Check whether the given D.E $xdy + ydx = 0$ is exact [2]
- b) Write the Bernoulli equation [2]
- c) Solve the D.E $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ [2]
- d) Find the P.I of $(D^2 + 4)y = \sin 2x$ [2]
- e) Test whether $e^x x \frac{\partial z}{\partial x} + x^2 y \frac{\partial z}{\partial y} = z$ is linear or nonlinear PDE [2]
- f) Write the PDE corresponding to $z = ax + by + \sqrt{a^2 + b^2}$ [2]
- g) Find $\text{curl } \bar{f}$ where $\bar{f} = xy^2\bar{i} + 2x^2yz\bar{j} - 3yz^2\bar{k}$ at the point $(1, -1, 1)$ [2]
- h) If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at $(1, -2, -1)$ [2]
- i) Define volume integral [2]
- j) Using Divergence theorem Evaluate $\iiint_s \bar{F} \cdot \bar{n} ds$ where s is the surface of [2]

the sphere $x^2 + y^2 + z^2 = b^2$ in the first octant where $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$.

PART-B

(50 Marks)

Unit-1

- 2 a) Solve the D.E $\left(\frac{y}{x} \sec y - \tan y\right) dx + (\sec y \log x - x) dy = 0$ [5]
- b) Solve the D.E $\frac{dy}{dx} - x^2y = y^2e^{-\frac{x^3}{3}}$ [5]

(OR)

- 3 a) Determine the charge and current at any time in t in R-C circuit with R = 10 ohms, c= 2 Farad and E = 100 volts given that $q(0) = 0$ [5]
- b) The temperature of a cup of coffee is 92°C , when freshly poured the room temperature being 24°C . In one minute, it was cooled to 80°C . How long a period must elapse, before the temperature of the cup becomes 65°C . [5]

Unit-2

- 4 a) Solve the D.E $(D^2 + a^2)y = \sec ax$ [5]
- b) Solve the D.E $(D^2 + 5D + 4)y = x^2 + 7x + 9$ [5]

(OR)

- 5 a) Solve the D.E $y^{11} - 2y^1 + y = e^x \log x$ using Method of variation of parameters [5]
 b) Determine the charge on the capacitor at any time $t > 0$ in circuit in series having an emf $E(t) = 100 \sin 60 t$, a resistor of 2 ohms, an inductor of 0.1 Henries and capacitor of $\frac{1}{260}$ farads, if the initial current and charge on the capacitor are both zero. [5]

Unit-3

- 6 a) From the differential equation of all planes which are at a constant distance 'a' from the origin [5]
 b) Solve the PDE $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$ [5]
 (OR)
 7 a) Form the differential equation by elimination arbitrary function from $\phi(xy + z^2, x + y + z) = 0$ [5]
 b) Solve the PDE $(D^4 - 2D^3D^1 + 2DD^{13} - D^{14})z = 0$ [5]

Unit-4

- 8 a) Prove that $\nabla(r) = \frac{\vec{r}}{r}$ [5]
 b) Find the directional derivative of $\phi = xy^2 + yz^2 + x^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at $(1, 1, 1)$. [5]
 (OR)
 9 a) Prove that $\text{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{curl} \vec{A} - \vec{A} \cdot \text{curl} \vec{B}$ [5]
 b) Prove that $\vec{f} \cdot \text{curl} \vec{f} = 0$ if $\vec{f} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$ [5]

Unit-5

- 10 a) Find the work done in moving particle in the field $\vec{F} = 3x^2 \vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$. [5]
 b) Using Green's theorem to evaluate $\oint_c (x^2 y dx + y^3 dy)$ where c is the closed path formed by $y = x; y = x^3$ from $(0, 0)$ to $(1, 1)$ [5]
 (OR)
 11 Evaluate (i) $\int_V \nabla \cdot \vec{F} dv$ and (2) $\int_V \text{Curl} \vec{F} dv$ for $\vec{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$ [10]
 along the Region bounded by the planes $x = 0; y = 0, z = 0$ and $2x + 2y + z = 4$.
